STRATIFICATION OF THE IONOSPHERE AND THE ORIGIN OF THE E_1 LAYER*

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ABSTRACT. Calculations of upper atmospheric ionisation are made after Pannekoek with certain modifications. It is assumed that the upper atmosphere above 100 km. consists mainly of N_2 and O while the lower and the middle atmosphere below this level consists of N_2 and O_2. There is a layer of transition between the two occupying the region 80-130 km. The temperature in the upper atmosphere is assumed to be 600°K and that in the transition layer 300°K. The sun is taken to be radiating like a black body at 6800°K. Calculations show that there are ionisation maxima at 250 km. due to O, at 160 km. due to N_2 and 90 km. due to O_2. These three maxima are identified with the E_2, E_1 and E_1 regions.

The most interesting point to be noted is that attempts at obtaining from theoretical considerations a layer of ionisation at the height of the E_1 region have hitherto failed. This, in fact, prompted Chapman to put forward the hypothesis that the ionisation of the F region was due to ultra-violet radiation while that of the E region was due to the bombardment of neutral corpuscles shot off from the sun. It is well-known that observations during solar eclipses prove definitely that the E_1 region ionisation is also due to solar ultra-violet radiation. The work described here thus removes the anomaly regarding the formation of the E_1 layer. It can now be definitely asserted that the region round 100 km. in which rapid transition of O_2 to O occurs is also, as a consequence, the region of maximum ionisation of O_2 and that the ionised layer formed near this level is to be identified with the E_1 layer.

INTRODUCTION

The ionosphere is now known to be stratified into a number of regions or layers of maximum ionisation. The layers of most persistent ionisation are in the upper atmosphere above 90 km. In recent years strong evidence [1] has been obtained of the occurrence of ionised regions in the middle (15-90 km.) and also in the lower (0-15 km.) atmosphere. The heights and the accepted nomenclature of these regions of ionisation are given in the table below.

	Equivalent height.	Nomenclature.
Lower atmosphere	0-15 km.	C_1 region
Middle atmosphere	15-45 ,,	C_2 ,,
	506-5 ,,	D ,,
Upper atmosphere	90-120 ,,	E_1 ,,
	150-170 ,,	E_2
	180-220 ,,	F
	220 km. upwards	F_2

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Of the layers in the upper atmosphere E_1 and F_2 are permanent features of the ionosphere and are found to exist at all times of the day and night throughout the year. E_2 and F_1 are generally present during the daytime. Compared with the layers in the upper atmosphere, those in the middle and the lower atmosphere are very irregular in occurrence In fact more recent observations[2] appear to show that the B and the C regions occur only occasionally as patches or blobs of ionisation which can reflect radio waves.

The origin of the stratifiction is still ill-understood though the agency causing the ionisation is now known more or less definitely to be the solar ultra-violet radiation. Besides the solar radiation a number of agencies of geophysical origin like thunderstorms, meteoric showers, etc., have also been suggested as producing ionisation, the former particularly in the lower regions of the ionosphere. Appleton[3] suggested in 1933 that the four uppermost regions are connected with the ionisation potentials of nitrogen and oxygen atoms and molecules, though he did not offer any definite explanation as to the mechanism of the stratification.

Pannekoek[4] in 1926 developed a method of computing upper atmosphere ionisation by assuming the sun as a black radiator and applying Woltjer's[5] extension of Saha's theory of thermal ionisation. Since in this theory the absorption of the ionising radiation by each of the constituents is considered separately it is to be expected that the different constituents should, in general, yield different distributions of ionisation with height. The stratification and other details of the ionosphere were unknown at the time of Pannekoek's work and the significance of the results of his analysis were not properly appreciated when his paper was published.

After the discovery of the two main stratifications of the ionosphere, E_1 and F_2, Chapman [6] in 1931 attempted to apply Pannekoek's analysis to explain the stratification by assuming the upper atmosphere to consist of N_2, O_2 and O and taking the temperature to be about 300°K. Chapman was unable to trace the origin of the ionisation maximum at the level of the E_1 region, *i.e.*, at about 100 km., to any known process of atomic or molecular ionisation. This result, as is well-known, led him to offer the hypothesis that the upper layer is due to the action of solar ultra-violet radiation and the lower one to the action of high speed neutral corpuscles shot off from the sun. Subsequent observations during solar eclipses,[7] however, failed to substantiate Chapman's theory. At the same time Chapman [8] developed his theory of ionisation by considering the absorption of monochromatic radiation in an atmosphere rotating with the earth. The results of his analysis have been widely utilised for studying the variation of ionisation at different hours of the day and night and in different latitudes. It has recently been shown by Saha and Rai[9] that Chapman's theory if it were 'properly developed and extended,' would have led to the same result as that of Pannekoek.

Since the publication of Chapman's work our ideas regarding the state and constituents of the upper atmosphere have undergone considerable modification. In view of this it is desirable that the problem of the stratification of the ionosphere be re-examined in the light of our newer knowledge regarding the upper atmosphere. This has been done recently by the writer both with regard to the formation of the upper F_1 and F_2 layers and the lower E_1 layer. A communication[10] on the former was made to the Mathematics and Physics section of the Indian Science Congress (January, 1938) and the results arrived there are described here after revised calculations. In the present paper the question of the E_1 layer ionisation is also discussed. In connexion with this latter it should be mentioned that it had not been hitherto possible to explain the ionisation at the level of the E_1 layer by the application of Pannekoek's theory. In a recent paper by Hulburt[11] in which he has utilised Pannekoek's formula in a modified form, no indication of ionisation below 200 km. was obtained. It will be shown below that this curious result of the analyses by the above workers, namely, no evidence of ionisation at the E_1 layer level, may be traced to the special nature of the assumptions made regarding the distribution of O_2. Hulburt has assumed that O_2 is distributed above 110 km. according to Dalton's law, the temperature of the atmosphere being 360°K. The distribution of O_2 assumed by Chapman has not been definitely mentioned. It is now believed, however, that due to the photo-dissociating effect of the solar radiation all the O_2 molecules above about 130 km. are completely dissociated to O. There is thus a layer of transition between, say, 80 km. and 130 km. in which O_2 is transformed to atomic oxygen. Consideration of the photo-ionisation of O_2 in this region of transition shows that an extremely thin ionised layer is formed with a maximum of ionisation at about 100 km. level *The definite assertion can thus be made that the 100 km. level which is the level of rapid transition of O_2 to O is also, as a consequence, the level of maximum ionisation of O_2 and the ionised layer formed in this region is to be identified with the E_1 layer.*

2. PANNEKOEK'S METHOD AND ITS MODIFICATION

As mentioned above Pannekoek's method of treating the ionisation of the upper atmosphere has been the earliest and is the most general method In our calculations of the distribution of ionisation due to the various gases, we will use Pannekoek's method* with certain minor modifications. We now proceed to discuss briefly the derivation of the working formula which we have actually employed in section 5.

* I am greatly indebted to Prof M. N. Saha for the kind help I received from him in clearing up a number of points in Pannekoek's theory and also for helpful criticisms regarding the contents of this paper. Fig. 1 depicting the absorptions by N_2 and by O_2 in the region $\lambda < 1000$A is drawn at his suggestion.

(a) Woltjer and Milne's extension of the Saha Ionisation Formula

The theory of the ionisation of elements by the heat was, as is well-known, first given by Saha[12] in 1921. The fraction of a hot gas which is ionised is given by

$$\frac{y^2}{1-y^2} \cdot p = \frac{(2\pi m_e)^{\frac{3}{2}}}{h^3} (k\mathrm{T})^{\frac{5}{2}} e^{-\chi/k\mathrm{T}} \quad \ldots (1)$$

where y is the fraction of the gas ionised,

p—the total pressure due to the ions, electrons and neutral atoms (dynes/cm^2.),

m_e—the mass of electrons (gms.),

T—the absolute temperature of the gas in the equilibrium state (degrees absolute),

h—Planck's constant,

k—Boltzmann's constant,

χ—energy of ionisation per atom (ergs).

The formula strictly holds when the gas, the electrons, the ions and the radiation are in perfect thermodynamical equilibrium. One however meets with difficulty if one tries to apply the formula to the case of the atmosphere of a star, say, that of the sun, which is at a temperature of about 500°K but which is traversed by radiation at a temperature as high as 6800°K emanating from the photosphere. The case might be compared to the laboratory experiments in which light from an arc at 4000°C emitting like a black body is made to traverse a column of sodium vapour at a temperature of 400° to 500°C. Since the case of ionisation of the earth's atmosphere belongs to this class—the temperature of the atmospheric gases being, say, 300° to 1500°K while the temperature of solar radiation is 6800°K—the Saha ionisation formula as given above in its original form cannot be applied. One has therefore to suitably modify the ionisation formula in order to apply it to the case under discussion.

A preliminary attempt in this direction was made by Saha and Sur[13] but the correct formula was later derived by Woltjer. The thermodynamical derivation of this formula is rather complicated. An easier deduction based on kinetic theory considerations was given by Milne [14] in 1924. Starting from this formula we shall proceed to compute the ionisation of the upper atmosphere according to the method indicated by Pannekoek introducing the necessary modifications at their proper places.

Woltjer showed that

$$\frac{n_i n_e}{N} = \left[\frac{n_i n_e}{N}\right]_0 \frac{\int_{\nu_0}^{\infty} \psi_\nu I_\nu d\nu}{\int_{\nu_0}^{\infty} \psi_\nu e^{-h\nu/kT}\left(I_\nu + \frac{8\pi h\nu^3}{c^2}\right)d\nu} \qquad \ldots \quad (2)$$

where n_i is the number of ions per unit volume,

n_e—number of electrons per unit volume,

N—number of atoms per unit volume,

ν—frequency of the radiation,

I_ν—intensity of the radiation of frequency ν,

ψ_ν—probability coefficient, it gives the fraction of the total number of atoms ionised when radiation of unit intensity and of frequency ν is incident upon them,

$\left[\frac{n_i n_e}{N}\right]_0$—value of $\frac{n_i n_e}{N}$ in the equilibrium condition, *i.e.*, when radiation is at the same temperature with the gas,

ν_0—threshold frequency for ionisation,

T—temperature of the gas in degrees absolute,

c—velocity of light.

(*b*) *Modification of the above Formula for Limited Range of Integration over which Photo-electric Absorption is assumed Constant*

The evaluation of the integrals involved in eqn (2) requires a knowledge of ψ_ν as a function of ν. Pannekoek has assumed ψ_ν to be proportional to ν^{-3}. It is known, however, that ψ_ν is connected with the atomic absorption coefficient γ_ν by the relation

$$\psi_\nu = \gamma_\nu / h\nu.$$

As will be seen later on we shall assume the absorption coefficient to be constant over a limited range of integration. It follows from this assumption that $\psi_\nu = A/\nu$ (where A is a constant) within these limits.

We may therefore write eqn. (2) as

$$\frac{n_i n_e}{N} = \left[\frac{n_i n_e}{N}\right]_0 \frac{\int_{\nu_0}^{\nu'} I_\nu/\nu \cdot d\nu}{\int_{\nu_0}^{\nu'} \frac{1}{\nu} \cdot e^{-h\nu/kT}\left(I_\nu + \frac{8\pi h\nu^3}{c^2}\right) d\nu}$$

where ν_0 and ν' are the limits between which the photo-electric absorption coefficient γ_ν is assumed to be constant.

Since, $e^{h\nu/kT_1} >> 1$, we may put

$$I_\nu = \frac{8\pi h\nu^3}{c^2} \cdot \frac{1}{e^{h\nu/kT_1}}$$

$$<< \frac{8\pi h\nu^3}{c^2}, \text{ where } T_1 \text{ is the temperature of the black body.}$$

Remembering these, the above eqn. becomes

$$\frac{n_i n_e}{N} = \left[\frac{n_i n_e}{N}\right]_0 \frac{\int_{\nu_0}^{\nu'} \frac{8\pi h\nu^2}{c^2} \cdot e^{-h\nu/kT_1} d\nu}{\int_{\nu_0}^{\nu'} \frac{8\pi h\nu^2}{c^2} \cdot e^{-h\nu/kT} d\nu}$$

$$= \left[\frac{n_i n_e}{N}\right]_0 T_1^3 \cdot T^{-3} \frac{\int_{x_{10}}^{x'} x_1^2 e^{-x_1} dx_1}{\int_{x_0}^{x'} x^2 e^{-x} dx}$$

where $x_1 = h\nu/kT_1$ and $x = h\nu/kT$.

Now it can be proved that

$$\frac{n_i n_e}{N} = \frac{y^2}{1-y^2} \cdot \frac{p}{kT},$$

y having its previous significance.

Taking this into account and substituting the value of $\left[\frac{n_i n_e}{N}\right]_0$ from the original Saha formula (eqn. 1), we get

$$\frac{y^2}{1-y^2}p = \frac{(2\pi m_e)^{\frac{3}{2}}}{h^3} \cdot k^{\frac{5}{2}} \cdot T^3 T^{-\frac{1}{2}} e^{-\chi/kT} \frac{\int_{x_{10}}^{x'} x_1^2 e^{-x_1} dx_1}{\int_{x_0}^{x'} x^2 e^{-x} dx} \quad \ldots (3)$$

where p is in dynes/cm.^{2} and χ is in ergs

The expression on the right hand side of this equation is different from that of Pannekoek. If, however, the values of the limits x_0 and x' are such that

$$\int_{x_0}^{\infty} x^2 e^{-x} dx \quad >> \quad \int_{x'}^{\infty} x'^2 e^{-x'} dx'$$

and that $x^2 >> x$

then it is easy to show that eqn. (3) takes the form

$$\frac{y^2}{1-y^2} \cdot p = \frac{(2\pi m_e)^{\frac{3}{2}}}{h^3} \cdot k^{\frac{5}{2}} T_1 \cdot T^{\frac{3}{2}} e^{-\chi/kT_1} \quad \ldots (4)$$

which is the same as that of Pannekoek's.

3. APPLICATION TO THE EARTH'S ATMOSPHERE.

In calculating upper atmospheric ionisation from eqn. (3) it is necessary, first of all, to find out what fraction of the total energy radiated by the sun per second enters into the earth's atmosphere. The value of this fraction is $\omega/4\pi = \beta$, say, where ω is the solid angle subtended by the sun at the earth. Chapman[6] and Saha and Rai[9] have taken $\beta = 1/230{,}000$; we shall also use this value. Hence we have to multiply the right hand side of eqn. (3) by β so that we get

$$\frac{y^2}{1-y^2} \cdot p = \beta \frac{(2\pi me)^{\frac{3}{2}}}{h^3} \cdot k^{\frac{5}{2}} T_1^{\frac{3}{2}} T^{-\frac{1}{2}} e^{-\chi/kT} \frac{\int_{x_{10}}^{x'} x_1^2 e^{-x_1} dx_1}{\int_{x_0}^{x'} x^2 e^{-x} dx} = K, \text{ say...} \quad (5)$$

Now the density of ionisation at any point due to the photo-ionisation of a particular gas depends upon the intensity of the effective radiation reaching that point. This intensity will, in its turn, depend upon the absorption suffered by the radiation in passing through the superincumbent atmosphere. This latter can be calculated if we know the coefficient of absorption γ of the gas per atom or molecule and also the distribution of the pressure of the gas with height in the atmosphere.

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where $x_1 = h\nu/kT_1$ and $x = h\nu/kT$.

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$$\frac{y^2}{1-y^2}p = \frac{(2\pi m_e)^{\frac{3}{2}}}{h^3} k^{\frac{5}{2}} \cdot T^3 T^{-\frac{1}{2}} e^{-\chi/kT} \frac{\int_{x_{10}}^{x'} x_1^2 e^{-x_1} dx_1}{\int_{x_0}^{x'} x^2 e^{-x} dx} \quad \ldots (3)$$

where p is in dynes/cm.^{2} and χ is in ergs.

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Now the density of ionisation at any point due to the photo-ionisation of a particular gas depends upon the intensity of the effective radiation reaching that point. This intensity will, in its turn, depend upon the absorption suffered by the radiation in passing through the superincumbent atmosphere. This latter can be calculated if we know the coefficient of absorption γ of the gas per atom or molecule and also the distribution of the pressure of the gas with height in the atmosphere.

Let us suppose that the density N_z of a gas at a height z above a certain datum level be given by

$$N_z = f(N_0, z) \quad \ldots (6)$$

where N_0 is the number of molecules or atoms per c.c. at the datum level.

Let us now consider an atmospheric layer of thickness dz at the height z and of unit cross-section. The absorption of the radiation in passing through this layer will obviously be given by

$$\frac{dI}{I} = \gamma N_z dz$$

where I is the intensity at incidence,

γ—the absorption coefficient per molecule or atom,

and z is reckoned positive upwards.

Substituting for N_z from eqn. (6), we get

$$\frac{dI}{I} = \gamma f(N_0, z) dz$$

Integration yields

$$I = I_0 e^{\gamma \int_0^{N_z} f(N_0, z) dz} \quad \ldots (7)$$

where I_0 is the intensity at the level at which N is zero, *i.e.*, outside the atmosphere of the absorbing gas.

The distribution of pressure generally follows the exponential law, *i.e.*,

$$N_z = f(N_0, z)$$
$$= N_0 e^{-\theta z}$$

where θ is a constant which determines the nature of the distribution. We then have

$$I = I_0 e^{-\gamma N_z / \theta}$$

It is evident, therefore, that to obtain the actual ionisation density produced by the incident radiation at the height z at which the density of the gas molecules or atoms is N_z the expression for K (eqn. 5) will have to be multiplied by the factor

$$e^{-\gamma N_z / \theta}$$

Thus

$$\frac{y^2}{1-y^2} \cdot p = K e^{-\gamma N_z / \theta} \quad \ldots (8)$$

4. UPPER ATMOSPHERIC CONSTITUENTS, AND THEIR TEMPERATURE AND ABSORPTION COEFFICIENTS ASSUMED FOR THE CALCULATIONS

In order to evaluate the right hand side of eqn. (8) it is necessary that we know the properties of the atmosphere we are considering. A knowledge of the temperature, constituents and pressure distribution in the upper atmosphere as well as the absorption coefficients and the limits of absorption of the constituents is necessary. We give below the assumptions we have made with regard to these and also our reasons for making the same.

(a) Upper Atmospheric Constituents and Temperature

As mentioned in the introduction our ideas regarding the temperature and the constituents of the upper atmosphere have in recent years undergone considerable modification. Leaving aside the older view prevalent before the identification of the auroral green line (of atomic oxygen) that the higher regions of the atmosphere consisted of the hypothetical element geocoronium, the belief that the main constituents of the upper atmosphere were hydrogen and helium persisted for a very long time. These gases were subsequently shown to be liable to escape from the atmosphere owing to insufficient pull of gravity. The view held till recently had therefore been that the upper atmosphere constituents were N_2 and O_2. Even this view is now losing ground and our latest conception regarding the upper atmosphere is that it consists of N_2 and O. The absence of O_2 is explained as due to the photo-dissociating action of the solar rays converting all the O_2 to O down to about 100 km. level. Since our calculations on the ionisation in the atmosphere will be based on the assumptions that the atmosphere below 100 km. consists mainly of N_2 and O_2, and that above, of N_2 and O and there is a layer of transition between the two round about 100 km. we would discuss these points in a little more detail.

The dissociation of O_2 to O is brought about by the absorption of the continuum of the Runge-Schumann bands beyond $\lambda 1750$A according to the following reaction

$$O_2 + h\nu = O\ {}^3P + O\ {}^1D_2$$

The first atom is in the normal state and the second one in an excited state. Now since the process of recombination depends upon the pressure, the oxygen atoms in the rare high atmosphere will be almost permanently free while those in the lower and the comparatively denser atmosphere will recombine and produce O_2. There will thus be a transition layer between the high and the middle atmosphere in which the amount of O_2 would rapidly increase with decreasing height while that of O would rapidly diminish till it is practically non-existent. Calculations carried

out recently by Majumdar,[15] though not quite rigorous, show that the transition occurs fairly sharply and that the atmosphere at and above about 130 km. is almost entirely devoid of O_2.

On the basis of the above we have assumed the following constitution of the atmosphere. From the ground level up to 80 km. the atmospheric constituents are principally N_2 and O_2 (except of course the ozonosphere between 20 and 45 km.) in a more or less mixed state. Between 80 and 130 km. the density of the O_2 molecules decreases rapidly while that of O increases. This decrease of the density of O_2 we assume to occur exponentially according to the equation

$$N_z = N_0 e^{-\theta z}$$

where z is the height reckoned from the 80 km. level, N_0 the density at this level and θ is a constant. The value of N_0 is known to a fair degree of approximation from radio and other meteorological observations and is taken as equal to $1{\cdot}6 \times 10^{14}$ O_2-molecules/c.c.[16] At 130 km. the number of O_2-molecules is greatly diminished, which we assume to be by a factor $0{\cdot}625 \times 10^{-10}$, so that, it is 10^4 molecules/c.c. These assumptions give the value of θ to be $0{\cdot}46 \times 10^{-3}$.

Above 130 km. we assume that the only gases present are N_2 and O and that these are in diffusive equilibrium under the action of gravity. In other words each of these constituents are distributed according to Dalton's law

$$N_z = N_0 e^{-mgz/kT} \text{ molecules/c.c.}$$

where N_0 is the density at the datum level (130 km.) in molecules/c.c.,
m—the mass of each molecule,
z—the height reckoned from the datum level,
and T—the atmospheric temperature at the levels concerned.

It should be noted that we have assumed above that the upper atmosphere consists mainly of N_2 and O. In other words, we have assumed that unlike O_2, N_2 is not photo-dissociated. This assumption is based on the fact that the forbidden lines of N are apparently absent in the light of the night sky and of the aurorae. According to Saha,[17] however, this apparent absence is not to be regarded as a criterion for the absence of N in the upper atmosphere. To quote his remarks,—

" The forbidden lines of nitrogen are

$$^4S - {}^2D = \nu \begin{cases} 19223 \\ 19231 \end{cases} \quad \begin{matrix} 5202{\cdot}1 \\ \lambda \\ 5199{\cdot}9 \end{matrix}$$

$$^4S - {}^2P = \nu 28840 \quad \lambda 3467{\cdot}4$$

$$^2D - {}^2P = \nu \begin{cases} 9617 \\ 9606 \end{cases} \quad \begin{matrix} 10398{\cdot}3 \\ \lambda \\ 10410{\cdot}2 \end{matrix}$$

The last is in the infra-red and has not yet been observed either in the morning or the evening flash. $^4S-{}^2P$, $\lambda 3467{\cdot}4$, is too far in the ultra-violet and has not yet been found. $^4S-{}^2D$, $\lambda 5202$, is promising, but it is unfortunately mixed up with the (0,2) bands of N_2^+. Sommer at one time identified the line observed near $\lambda 5206$ as being the $^4S-{}^2D$ line of N, but the matter needs careful reinvestigation. The discussion shows that as yet there is no definite evidence for the presence of forbidden lines of N in the night-sky spectrum, but this does not absolutely exclude the possibility of their occurrence."

Since the above was written by Saha, Kaplan[18] and also Bernard[19] simultaneously reported the existence of a line $\lambda 3470$ in the spectra of the light of the night sky and of aurorae. This has been provisionally identified with the line $^1S-{}^2P$, the wave-length of which is $\lambda 3467{\cdot}4$A. In a private letter to Prof. Saha, Prof. Kaplan informs him that his new measurement with an apparatus of higher dispersion gives the wave-length as $\lambda 3467$A. It seems, therefore, that the identification is satisfactory and that one cannot altogether deny the existence of atomic nitrogen in the upper atmosphere However, in the absence of more detailed knowledge regarding the presence and distribution of atomic nitrogen we have in the present communication thought fit to omit the consideration of this gas in subsequent discussions.

It may also be noted here that the hypothesis that the upper atmosphere consists chiefly of N_2 and O raises a difficulty with regard to the explanation of the production of aurorae at great heights—1000 km. and above. The spectra of the aurorae give unmistakable evidence of the presence of molecular nitrogen but it is difficult to see how N_2 molecules can reach such great heights. On account of diffusive separation between N_2 and O, the former will occupy the comparatively lower regions of the upper atmosphere and the highest regions will consist mainly of atomic oxygen. This difficulty, which is a serious one and to which attention has been drawn by several workers, can be met with if we accept the auroral hypothesis developed recently by Hulburt and Maris.[20] According to these authors nitrogen molecules producing the aurorae do not attain the auroral heights from the denser atmosphere directly below the aurorae but are transported to the auroral regions from the lower atmosphere above the equatorial regions by being ionised by the solar rays and guided by the terrestrial magnetic field.

The temperature of the high atmosphere which we are concerned with is still a matter of speculation. Anomalous variations in the electron density of the F region of the ionosphere lend support to the hypothesis of a high temperature in this region. Martyn and Pulley[16] think that the temperature between 100 km. and 300 km. level rises linearly from 300° to 1200°K. In view of the still doubtful nature of this too definite assumption we have taken the temperature to be uniform and 600°K. The temperature between 80 and 130 km. in the transition layer ($O_2 \rightarrow O + O$)is taken to be 300°K. Consideration in regard to the application of Pannekoek's method for calculating the ionisation

distribution is also one of the reasons for assuming a constant temperature. In the case of the temperature varying with height, as Wildt [21] has shown, the theory of thermodynamical equilibrium no longer holds.

In table 1 the assumptions regarding the composition, pressure distribution and temperature described above are summarised.

TABLE 1.

Region.	Mean Temp. T.	Constituents.	Density Distribution.
80 km. to 130 km.	300°K	O_2, N_2, and O	For O_2 $N_z = N_0 e^{-\theta z}$ $N_0 = 1{\cdot}6 \times 10^{14}$ molecules/c.c. $\theta = 4{\cdot}6 \times 10^{-6}$ Datum level – 80 km.
130 km. upwards	600°K	O and N_2	$N_z = N_0 e^{-mgz/kT}$ $N_0 \begin{cases} = 4 \times 10^{11}$ atoms/c.c. for O $= 1{\cdot}6 \times 10^{12}$ molecules/c.c. for N_2
			Datum level – 130 km.

The values of N_0 for N_2 and O at 130 km. are calculated from the data given in a recent paper by Mitra and Rakshit [22] on the distribution of the constituent gases in the upper atmosphere.

(b) *The Coefficients of Absorption*

In section 3 we have seen how the ionisation formula (eqn. 2) should be modified when one is to calculate the ionisation of a gas of which the absorption coefficient is constant within certain frequency limits. Our knowledge regarding the absorption coefficients of the constituents which we are considering are extremely meagre. All previous workers on this problem have utilised the well-known formula of Kramers [23] developed for X-ray absorption :

$$\gamma_\nu = \frac{16\pi^2}{3\sqrt{3}} \cdot \frac{Z^2 e^6}{c} \cdot \frac{\nu_0}{(h\nu)^3}$$

We will also do the same* for O_2 and N_2. For atomic oxygen we will utilise the value of the absorption coefficient recently obtained by Saha and Rai[9] from wave mechanical considerations.

It will be seen that Kramers' formula involves the atomic number Z. When applied to the case of molecules like N_2 and O_2, Z cannot obviously be identified with the ordinary atomic number on account of the effect of " screening " of the nuclear charge due to the presence of the outer electrons. To compensate for the screening effect the calculated value of γ_{ν_0} (for $\nu=\nu_0$) has to be multiplied by a fraction. The magnitude of this fraction can be estimated by comparing the value of the absorption coefficient for O as deduced from Kramers' formula with that deduced quantum mechanically by Saha and Rai. This fraction is found to be (Saha and Rai : Kramers) 1/9 for the process $O^3P \longrightarrow O^1S$. It will not be unreasonable to assume that the fraction is of the same order for O_2 and N_2. On account, however, of the uncertainty and also the large range of values possible we have carried out our calculations by assuming this fraction to be 1/10 and 1/50, so that if at any future date more exact values of the absorption coefficient be available either experimentally or from theoretical considerations it may be possible to identify the results of one or other of the above sets of calculations as the actual ionisation distribution.

(c) *Limits of Absorption*

The limits of the wave-length ranges absorbed by the gases with which we are concerned may be obtained from spectroscopic data. Our knowledge regarding the absorption of ultra-violet light by N_2, O_2 and O has been collected by Saha,[17, 26] and by Chapman and Price.[27] For convenience, we present here the collected data only for wave-lengths $\lambda < 1000$A, in the accompanying table, since it appears that wave-lengths above this value cannot appreciably ionise any of the above atmospheric gases.†

* The applicability of Kramers' law to the cases in question is open to serious objection,[9] though till recently it was being used by various authors in connection with astrophysical problems. Recently T. L. Page[24] of Oxford has carried ont experiments to test the validity of this law in the case of the hydrogen atom and has concluded from his results that the use of Kramers' law in astrophysical problems is unjustified A very interesting discussion in this connection in which several eminent astrophysicists took part has been recently published in " The Observatory." [25]

† There is a possibility that atoms or molecules in some of the excited states may be photo-ionised by light of frequency less than that corresponding to the observed ionisation potentials. But this possibility is left out of consideration in the present discussion.

TABLE 2.

	Spectral Region.	Phenomena observed.	Remarks.
Molecular Nitrogen	λλ1000-795A	Rydberg sequence of absorption bands converging to the limit λ795A. A few tenths of a mm. at N.T.P. cause almost complete absorption.	First observed by Price and recently by Worley and Jenkins.[28]
	λ<795A	Continuous absorption commencing at λ795A (15·5·3 e. v. . Each quantum absorbed produces N_2^+ *normal* and electron. A few tenths of a mm. at N T.P. show the continuum.	Observed by Worley and Jenkins [28]
	λλ795 661 A	Rydberg sequence of absorption bands converging to the limit λ661A. A few hundreths of a mm. at N. T P. suffice to bring out the absorption spectra.	Observed by Hopfield [29] and recently by Takamine and others.[30]
	λ<661A	Continuous absorption commencing at λ661A (18·67 e.v.). Each quantum absorbed produces N^+ *excited* and electron. Absorption same as above	"
Molecular Oxygen	λλ1300-1019A	Series of diffuse absorption bands tending to the limit λ1019A. A few tenths of a mm. at N. T. P. prodnce complete absorption	Observed by Price and Collins. [31]
	λ<1019A	An expected continuum of absorption commencing at λ1019A (12 . e. v) Each quantum absorbed should produce O_2^+ *normal* and electron Very weak absorption.	Not yet definitely observed
	λλ1000-670A	(*i*) Complicated system of absorption bands (I1, I,.....) which may be fitted into a Rydberg sequence converging to the limit λ764.7A (16 14 e.v). (*ii*) Another system of bands (M, N,.....) forming Rydberg sequence converging to the limit λ744A (16.5 e v). (*iii*) A third system of bands (P Q, ...) forming Rydberg sequence passing to the limit λ670A (18 25 e v) (*iv*) An ionisation continuum corresponding to the second system (M, N, ...) of bands starting abruptly at about λ740A (16.7 e.v.). Absorption in all the above cases is very strong, a few hundredths of a mm. at N T. P. being sufficient to show the bands clearly.	Observed by Hopfield and by Price and Collins [31]

TABLE 2 (*contd.*).

	Spectral Region.	Phenomena observed.	Remarks.
Atomic Oxygen	$\lambda<910$A	Each quantum absorbed leads to the reaction : $O^3P \longrightarrow O^{+}{}^4S$ (13.55 e.v.)	The intensity of absorption is not known experimentally. Absorption coefficient calculated theoretically by Saha and Rai is $2{\cdot}18\times10^{-17}$ cm.^{2} per atom.
	$\lambda<858$A	$O^1S \longrightarrow O^{+}{}^2P$ (14·4 e v.)	Effect negligible
	$\lambda<744$	$O^1D \longrightarrow O^{+}{}^2P$ (16.5 e.v.)	,,

In fig. 1 the data collected in the above table are represented graphically.

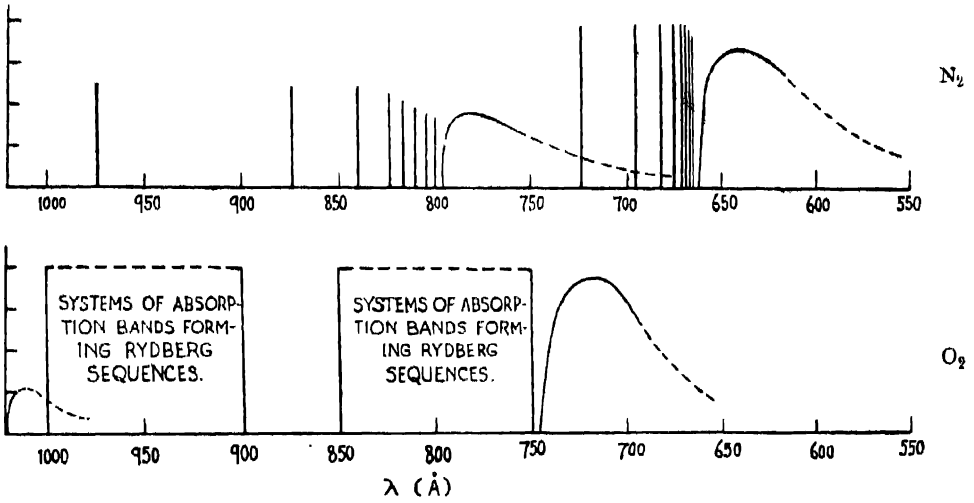

Fig. 1 – Absorption spectra of N_2 and O_2. The vertical lines represent the positions of the observed absorption bands while the heights of these lines represent the relative intensities of absorption approximately.

The ionisation of any of the constituent gases may, from considerations of the data given above, be regarded as the result of absorption of light within a particular range of wave-lengths, *e.g.*, λλ910-744A for atomic oxygen, λλ744-661A for molecular oxygen and $\lambda < 661$A for molecular nitrogen. The lower wave-length limit of absorption of nitrogen is not known but for purposes of calculation we shall take it as λ585A.

In table 3, the limits of the absorption ranges, the coefficients of absorption γ_{ν_0} and the ionisation potentials of N_2, O_2 and O arrived at in the light of the above considerations and used in our calculations are summarised.

TABLE

	Limit of absorption	Logarithm of coefficient of absorption.	Ionisation potential.
O	λλ910-744A $\nu\nu 3{\cdot}314\ 4\ 036 \times 10^{15}$	$\bar{1}7{\cdot}4487$	13·55 (e. v.)
O_2	λλ744·661A $\nu\nu 4{\cdot}036\text{-}4{\cdot}568 \times 10^{15}$	(*i*) $\bar{1}7{\cdot}2344$ (*ii*) $\bar{1}8{\cdot}5354$	16·5 (e. v.)
N_2	λλ661-585A $\nu\nu 4{\cdot}568\text{-}5{\cdot}128 \times 10^{15}$	(*i*) $\bar{1}8{\cdot}9852$ (*ii*) $\bar{1}8{\cdot}2862$	18·67 (e. v.)

In the actual calculations it is necessary that we take into account the variationo f γ_ν with frequency. In the case of O_2 and N_2, this variation follows, according to our assumption, Kramers' law, *i.e.*, γ_ν varies as $1/\nu^3$. In the case of O, according to Saha and Rai,[9] γ_ν varies as $\left(\frac{4}{\nu^3} - \frac{1}{\nu^4}\right)$. If these variations are taken into account over the ranges we are dealing with it is found (see Appendix) that the mean value of γ is of the order of $0{\cdot}8\gamma_{\nu_0}$. Our calculations have been made with these mean values of the absorption coefficients, treating them as constants throughout the entire ranges of integration.

5. RESULTS OF CALCULATIONS

The values of K for O, O_2 and N_2 are calculated from eqn. (3) with the proper limits of the integrals given in table 3. It should be noted that for the particular values of the limits dealt with, eqn. (3) approximates, as pointed out in section 2 (*b*), very closely to Pannekoek's equation.

(*a*) *Ionisation due to O*

It may be seen in table 2 that the radiation in the range λλ910-744A which is responsible for the photo-ionisation of O may also be partly absorbed by N_2. This latter absorption will evidently reduce the intensity of the radiation available for the ionisation of O and hence the ionisation density due to O will be less than what it would have been in the absence of N_2. The reduction, however, will not be very great since the relative density of the N_2 molecules at the heights at which the ionisation of O is produced is much smaller than that of O atoms. Lacking definite knowledge of the intensity of absorption by N_2 in this particular band, we shall assume that the radiation in this band is reduced to 0·05 of its original

value on this account. Hence, to calculate the ionisation due to O, we have to multiply the right hand side of eqn. (8) by the factor 0·05. Thus we have

$$\frac{y^2}{1-y^2} \cdot p = 0{\cdot}05\ \mathrm{K}\ e^{-\gamma \mathrm{N}_z/\theta}$$

Remembering that n_e, the number of electrons or ions* per c. c., is given by $\mathrm{N}_z y$, that $y^2 << 1$ and that $p = \mathrm{N}_z kT$ we get

$$n_e = \sqrt{\frac{0{\cdot}05\,\mathrm{K}\mathrm{N}_z}{kT}} \cdot e^{-\frac{1}{2}\cdot\gamma \mathrm{N}_z/\theta}$$

We have already assumed (Table 1) that the distribution of either of the gases N_2 and O obeys Dalton's law above 130 km. level. Hence the value of θ is mg/kT, where m is the mass of the oxygen atom.

The above equation gives the ionisation density corresponding to O at the height z at which the number of atoms per c. c. is N_z

(*b*) *Ionisation due to* O_2

The wave-length range responsible for the ionisation of O_2 is $\lambda\lambda$744-661A. There is, however, absorption by N_2 and O near this range of wave-lengths commencing from λ800A and from λ744A, respectively. We have assumed that O_2 exists in the lower and in the middle atmosphere and also that there is a transition layer in which O_2 is gradually replaced by O between 80 and 130 km. The ionising wave-lengths (for O_2) $\lambda\lambda$744-661A will, therefore, be partly absorbed by the overlying N_2 and O before reaching the transition layer. The value of the absorption coefficient of O is not known. Our only knowledge regarding the absorption by N_2 is that a few hundredths of a mm. of nitrogen at N.T.P. suffice to show the bands commencing at λ800A. The amount of N_2 and O above 130 km. being about 0·9 and 0·45 mm. at N. T. P, it would not be very wide of the mark if we assume that the intensity of the ionising radiation reaching the transition layer is reduced by a factor 0·001. This reduced value of the intensity of the incident radiation is used in our calculations. We thus have

$$\frac{y^2}{1-y^2} \cdot p = 0{\cdot}001\ \mathrm{K}\ e^{-\gamma \mathrm{N}_z/\theta}$$

And as before

$$n_e = \sqrt{\frac{0{\cdot}001\,\mathrm{K}\mathrm{N}_z}{kT}} \cdot e^{-\frac{1}{2}\gamma \mathrm{N}_z/\theta}$$

where $\theta = 4{\cdot}6 \times 10^{-6}$ (see table 1).

* We are dealing with unicomponent systems, so that $n_i = n_e$.

(c) *Ionisation due to* N_2

Radiation of wave-lengths in the range $\lambda\lambda < 661A$ which can ionise N_2 is not known to be absorbed by any of the above gases. In this case, therefore, we have simply

$$n_e = \sqrt{\frac{K N_2}{k'T}}\; e^{-\frac{1}{2}\gamma N_2/\theta}.$$

where $\theta = mg/k'T$, m being the mass of the N_2 molecule.

We now tabulate the values of K, γ, T and θ for the respective constituents.

TABLE 4.

	O	O_2	N_2
$\log_{10}K$	8·616	11·4	12·2316
$\log_{10}\gamma$	17·3348	(*i*) 17·1597 (*ii*) 18·4607	(*i*) 18·9053 (*ii*) 18·2063
T	600°K	300°K	600°K
$\log_{10}\theta$	7·5010	6·6628	7·7441

With the help of the above data, the values of n_e for different values of N_2 are calculated for each of the constituents and for each set of values of γ for O_2 and N_2. The height at which a particular value of N_2 occurs is obtained from the data given in table 1. The results are tabulated below.

TABLE 5.

(*a*) *Ionisation Distribution due to Atomic Oxygen*

N (Molecules per c. c.).	Height above ground (Km).	Ionisation density (Electrons/c. c.).
$10^{11\cdot5}$	155	$10^{2\cdot99}$
10^{11}	191	$10^{5\cdot94}$
$10^{10\cdot5}$	228	$10^{6\cdot70}$
10^{10}	264	$10^{6\cdot77}$
10^{9}	336	$10^{6\cdot41}$
10^{8}	408	$10^{5\cdot92}$
10^{6}	554	$10^{4\cdot92}$
10^{4}	700	$10^{3\cdot92}$

TABLE 5 (*contd.*).

(*b*) *Ionisation Distribution due to* O_2

N (Molecules per c.c.).	Height above ground (Km.).	Ionisation density (Electrons/c.c.)	
		(*i*) $\gamma = 10^{17\cdot1597}$.	(*ii*) $\gamma = 10^{18\cdot4607}$.
$10^{13\cdot6}$	83·5		$10^{2\cdot49}$
10^{13}	86	Very small	$10^{5\cdot18}$
10^{12}	91	$10^{5\cdot37}$	$10^{5\cdot91}$
10^{11}	96	$10^{5\cdot48}$	$10^{5\cdot53}$
10^{10}	101	$10^{5\cdot04}$	$10^{5\cdot04}$
10^{9}	106	$10^{4\cdot54}$	$10^{4\cdot54}$
10^{8}	111	$10^{4\cdot04}$	$10^{4\cdot04}$
10^{6}	121	$10^{3\cdot04}$	$10^{3\cdot04}$
10^{4}	131	$10^{2\cdot04}$	$10^{2\cdot04}$

(*c*) *Ionisation Distribution due to* N_2

N (Molecules per c.c.).	Height above ground (Km.).	Ionisation Density Electrons/c.c.).	
		(*i*) $\gamma = 10^{18\cdot9053}$	(*ii*) $\gamma = 10^{18\cdot2063}$.
$10^{12\cdot4}$	130		$10^{5\cdot27}$
10^{12}	140	$10^{7\cdot51}$	$10^{6\cdot03}$
$10^{11\cdot5}$	167	$10^{5\cdot41}$	$10^{6\cdot21}$
10^{11}	187	$10^{5\cdot84}$	$10^{6\cdot09}$
$10^{10\cdot6}$	208	$10^{5\cdot81}$	...
10^{10}	229	$10^{5\cdot83}$	$10^{5\cdot65}$
10^{9}	272	$10^{5\cdot10}$	$10^{6\cdot16}$
10^{8}	314	$10^{4\cdot66}$	$10^{4\cdot66}$
10^{6}	398	$10^{3\cdot66}$	$10^{3\cdot66}$
10^{4}	482	$10^{2\cdot66}$	$10^{2\cdot66}$

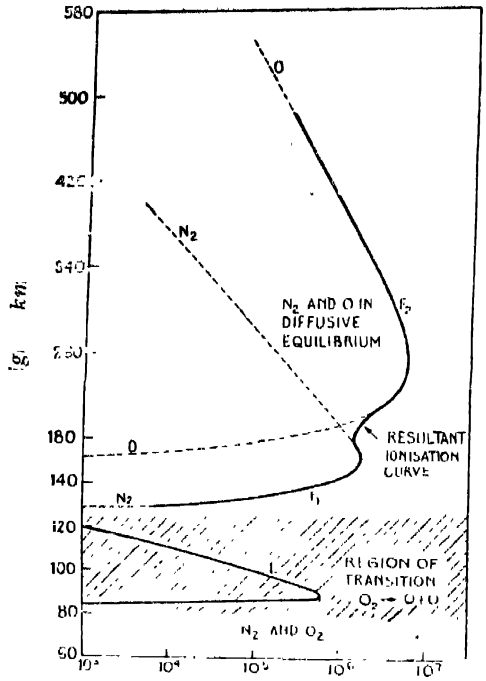

Fig. 2—Distribution of ionisation with height. $\gamma = 1/50\ \gamma$(Kramers).

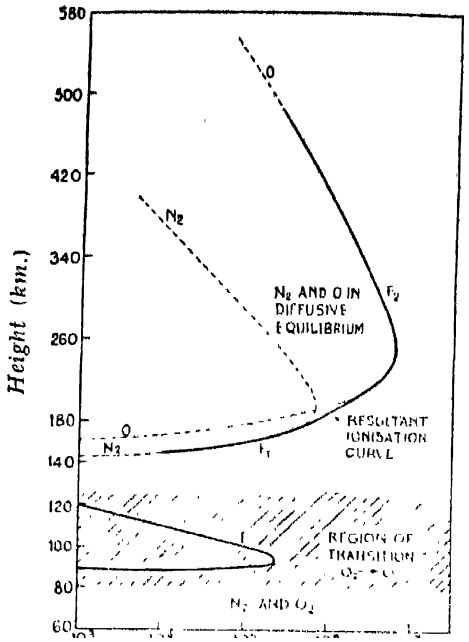

Fig. 3—Distribution of ionisation with height. $\gamma = 1/10\ \gamma$(Kramers).

The results are plotted in figs. 2 and 3.

DISCUSSION OF RESULTS

As mentioned in section 4(*b*), on account of the uncertainty regarding the value of γ for O_2 and N_2, we have assumed two possible different values of γ for these two gases. In figure 2 the values of γ for both O_2 and N_2 are taken as 1/50 of the corresponding values obtained from Kramers' formula on the assumption that the value of Z (effective nuclear charge) for O_2 and N_2 are 8 and 7, respectively. It will be seen that the upper thick line curve in figure 2 which is the resultant of the two curves due to O and N_2 (obtained by adding the abscissae) shows two distinct ionisation maxima at the levels of 250 and 160 kms. approximately. The upper maximum evidently corresponds to region F_2 and the lower one to region F_1. The respective electron densities are $6{\cdot}3 \times 10^6$ and $1{\cdot}6 \times 10^6$ per c.c. These values are to be regarded as those for equinoxial noon over the equator, *i.e.*, when the sun is directly overhead. With the variation of the solar activity the values of the ionisation density of all the layers will of course change enormously but it will be seen that the calculated values are of the correct order and also that the relative values of the maximum ionisation of the different layers as well as their heights agree closely to those actually observed.

The most interesting result of our calculations is with regard to the layer at about 100 km. level. There is no difficulty in identifying this with the E_1 layer the presence of which could not be hitherto explained theoretically. As mentioned in the introduction the position as well as the distribution of ionisation of this layer is determined by the existence of the region of transition of O_2 to O, the influence of which on the production of a layer of ionisation had not been considered till now. It will be noted that the layer is extremely thin; this is in conformity with experimental evidence. The calcutated value of the ionisation density ($6{\cdot}3 \times 10^5$ electrons/c.c.) seems to be slightly in excess of the observed value, but this may possibly be due to the fact that the reduction of intensity in the band $\lambda\lambda 744$-661A caused by the absorption of the overlying N_2 and O is greater than that assumed in section 5(*b*).

In figure 3, the values of γ for both O_2 and N_2 are taken to be 1/10 of γ(Kramers). The resultant ionisation curve for the upper region (F layer) does not show the two stratifications F_1 and F_2. Which of the two sets of values of the absorption coefficients [1/50 or 1/10 of γ(Kramers)] is more probable can only be decided by future experimental work on extreme ultra-violet absorption of these gases. It is to be noted that neither the height nor the distribution of ionisation of the layer at 100 km. (E_1 layer) is markedly affected by the assumed changes in the value of γ.

Without going through the whole series of calculations one can estimate how the position of the ionisation maximum will depend on the various factors, such as the temperature, the absorption coefficients and the pressure or density at the datum level.

The height at which the maximum ionisation for a given gas will occur is given by

$$z_{max} = H \log_e N_0 H\gamma$$

where $H = k'T/mg$, the "scale height."

Inspection of this formula shows that if γ is decreased the height will decrease. Thus if for the F_2 and F_1 layers, keeping γ for O constant, we decrease the value of γ for N_2, the effect will be to produce a greater separation of the two maxima.

An increase of temperature of the atmosphere will involve a rise in the levels of maximum ionisation.

Further if the proportion of nitrogen molecules with respect to atomic oxygen at the datum level be smaller than that assumed before (*i.e.*, if N_0 for N_2 is smaller than the assumed value, N_0 for O being constant) the stratification will be more pronounced because the level of maximum ionisation for N_2 will be lowered down.

In the case of the ionisation due to O_2, we have

$$z_{max} = \frac{1}{\theta} \log_e \frac{N_0 \gamma}{\theta}.$$

The value of N_0, the molecular density at the 80 km. level is known with fair accuracy. The value of θ will, however, vary with the assumed density at 130 km., where dissociation of O_2 is supposed to be practically complete. In our calculations we have taken the density at this level as 10^4 molecules/c.c., so that $\theta = 4{\cdot}6 \times 10^{-6}$. If instead of 10^4 we take 10^8 molecules/c.c., θ becomes equal to $2{\cdot}27 \times 10^{-6}$ and hence the level of maximum ionisation rises to a height of about 110 km. for both the values of γ.

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APPENDIX

(Reference page 17)

We have, from Kramers' law,

$$\gamma_{\nu_0} = \gamma_\nu \frac{\nu_0^3}{\nu^3}$$

mean value of γ between the limits ν_0 and ν' is given by

$$\gamma = \frac{\gamma_{\nu_0} \int_{\nu_0}^{\nu'} \frac{\nu_0^3}{\nu^3} d\nu}{\nu' - \nu_0}$$

$$= \frac{\gamma_{\nu_0} \cdot \nu_0}{2} \left[\frac{\nu' + \nu_0}{\nu'^2} \right] \qquad \ldots (a)$$

For O_2, $\nu_0 = 4{\cdot}036 \times 10^{15}$ and $\nu' = 4{\cdot}568 \times 10^{15}$ (Table 3). Hence from eqn. (*a*),

$$\gamma = 0{\cdot}83\ \gamma_{\nu_0}.$$

For N_2, $\nu_0 = 4{\cdot}568 \times 10^{15}$ and $\nu' = 5{\cdot}128 \times 10^{15}$. Hence $\gamma = 0{\cdot}84\ \gamma_{\nu_0}$

For O, we have[9]

$$\gamma_\nu = \frac{\gamma_{\nu_0}}{3}\left(\frac{4\nu_0^3}{\nu^3} - \frac{\nu_0^4}{\nu^4}\right)$$

$\therefore$ mean value of γ between the limits ν_0 and ν' is given by

$$\gamma = \frac{\gamma_{\nu_0}}{3}\frac{\int_{\nu_0}^{\nu'}\left(\frac{4\nu_0^3}{\nu^3} - \frac{\nu_0^4}{\nu^4}\right)d\nu}{\nu' - \nu_0}$$

$$= \frac{\gamma_{\nu_0}}{3}\left[\frac{5}{3} - \frac{\nu_0}{\nu' - \nu_0} - \frac{2\nu_0^3}{\nu'^2(\nu' - \nu_0)} + \frac{\nu_0^4}{3\nu'^3(\nu' - \nu_0)}\right] \quad \ldots \quad (b)$$

And for O, $\nu_0 = 3{\cdot}314 \times 10^{15}$ and $\nu' = 4{\cdot}036 \times 10^{15}$, so that we get, from eqn (*b*),

$$\gamma = 0{\cdot}77\ \gamma_{\nu_0}.$$

REFERENCES

1 Mitra and Bhar, *Science and Culture*, **1**, 782 (1936), Colwell and Friend, *Nature*, **137**, 782 (1936); Watson Watt and others, *Nature*, **137**, 866 (1936).

2 Appleton and Piddington, *Proc. Roy. Soc.*, A, **164**, 467 (1938).

3 Appleton, *Nature*, **131**, 172 (1933).

4 Pannekoek, *Proc. Amsterdam Akad.*, **29**, 1165 (1926).

5 Woltjer, *Physica*, **5**, 406 (1925).

6 Chapman, *Proc. Roy. Soc.*, A, **132**, 353 (1931).

7 Chapman and Appleton, *Proc. I.R.E.*, **23**, 658 (1935).

8 Chapman, *Proc. Phys. Soc.*, **43**, 26 (1931).

9 Saha and Rai, *Proc. Nat. Inst. Sci. India*, **4** 319 (1938).

10 Bhar, Paper No 58, Mathematics and Physics Section, Indian Science Congress (January, 1938).

[11] Hulburt, *Phys. Rev.*, **53**, 544 (1938).

[12] Saha, *Phil. Mag.*, **40**, 772 (1920).

[13] Saha and Sur, *Phil. Mag.*, **1**, 1025 (1926).

[14] Milne, *Phil. Mag.*, **47**, 209 (1924).

[15] Majumdar, *Ind. J. Phys.*, **21**, 75 (1938).

[16] Martyn and Pulley, *Proc. Roy. Soc., A*, **154**, 455 (1936); Ghose, *M.Sc. Thesis* (1933), *Calcutta University*.

[17] Saha, *Proc. Roy. Soc., A*, **160**, 155 (1937).

[18] Kaplan, *Nature*, **141**, 1139 (1938).

[19] Bernard, *Nature*, **141**, 1140 (1938).

[20] Hulburt and Maris, *Rev. Mod. Phys.*, **9**, 44 (1937).

[21] Wildt, *Astro. Jour.*, **83**, 1 (1936).

[22] Mitra and Rakshit, *Indian Jour. Phys.*, **12**, 47 (1938)

[23] Kramers, *Phil. Mag.*, **46**, 836 (1923).

[24] Page, *Nature*, **141**, 1137 (1938).

[25] *The Observatory*, **61**, 178 (1938).

[26] Saha, *Proc. Nat. Inst. Sci. India*, **1**, 217 (1935).

[27] Chapman and Price, *Reports on Progress in Physics*, **3**, 42 (1937).

[28] Worley and Jenkins, *Phys. Rev.*, **54**, 305 (1938).

[29] Hopfield, *Phys. Rev.*, **36**, 789 (1930).

[30] Takamine, Suga and Tanaka, *Sc. Pap. I.P.C.R.*, **34**, 854 (1938).

[31] Price and Collins, *Phys. Rev.*, **48**, 714 (1935).